SIMULATION OF NONLINEAR VISCO-ELASTIC BEHAVIOUR OF YARNS USING EYRING MODEL

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Abstract: *Eyring's non-linear visco-elastic model has been used to describe two important time dependent mechanical behaviors for textile materials such as stress relaxation and creep. Polyester–wool–lycra yarn spun from Siro spinning technology suitable for the production of stretch fabrics are considered in this study. This worsted core-spun yarn is chosen because they are suitable for the production of stretch fabrics for which stress relaxation and creep behaviors have great practical significance. The complex mathematical equations of Eyring's model are handled by a non-traditional evolutionary algorithm such as genetic algorithm. The findings show that Eyring's model not only simulates both stress relaxation as well as creep behaviors of the experimental yarns with reasonable degree of accuracy, but also deciphers the underlying molecular mechanism of the two behaviors for these yarns.*

Keywords: *Creep, Eyring's model, Genetic algorithm, Stress relaxation, Visco-elastic, Yarn.*

1. Introduction

The mechanical properties of a textile material are the time dependent phenomena because of its visco-elastic nature. Stress relaxation and creep are the two important time dependent mechanical behaviors of yarns manifesting the visco-elasticity. One way of analyzing the time dependent mechanical behavior is to use linear visco-elastic models composed of one or several elements such as ideal viscous dashpots obeying Newton's law of viscosity and ideal elastic springs obeying Hook's law. Maxwell and Voigt-Kelvin models are such types of models which consist of a single spring and a single dashpot in series and parallel respectively. However, neither of these models is adequate in explaining the general behavior of a visco-elastic material where it is necessary to describe both stress relaxation and creep. In this work Eyring's non-linear visco elastic model has been used to describe stress relaxation and creep behaviour. The most attractive feature of the Eyring's model is that it offers the possible identification of molecular mechanism and hence helps in unravelling some aspects of structure-dependence of mechanical behavior. The complex mathematical equations of Eyring's model are handled by a non-traditional evolutionary algorithm such as genetic algorithm.

2. Eyring's Visco-Elastic Model

Eyring's three elements visco-elastic model is shown in Fig 1. For the spring in the right hand arm of the model, the stress-strain relationship is given by

$$\varepsilon_1 = \frac{\sigma_1}{E_1} \tag{1}$$

where ε_1, σ_1 and E_1 are the strain, stress and the modulus of the spring respectively. Differentiating Eq. (1) we have

$$\frac{d\varepsilon_1}{dt} = \frac{1}{E_1}\frac{d\sigma_1}{dt} \tag{2}$$

For the dashpot, the strain rate of the non-Newtonian fluid is represented by the hyperbolic-sine law of viscous flow as follows

$$\frac{d\varepsilon_2}{dt} = A\sinh\alpha\sigma_1 \tag{3}$$

where ε_2 and σ_1 are the strain and stress of the dashpot respectively. A and α are the two constants of the non-Newtonian fluid. Constant A and $\alpha$are the indirect measures of activation free energy and activation volume of the flow respectively. As the spring and dashpot are in series, the total strain of the right arm is given by

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{4}$$

Differentiating Eq. (4) we have

$$\frac{d\varepsilon}{dt}=\frac{d\varepsilon_1}{dt}+\frac{d\varepsilon_2}{dt} \tag{5}$$

Substituting the relations from Eqs. (2 − 3), Eq. (5) becomes

$$\frac{d\varepsilon}{dt}=\frac{1}{E_1}\frac{d\sigma_1}{dt}+A\sinh\alpha\sigma_1 \tag{6}$$

For the spring in the left hand arm of the model, the stress-strain relationship is given by

$$\varepsilon=\frac{\sigma_2}{E_2} \tag{7}$$

where ε, σ_2 and E_2 are the strain, stress and the modulus of the left hand spring respectively. Differentiating Eq. (7) we have

$$\frac{d\varepsilon}{dt}=\frac{1}{E_2}\frac{d\sigma_2}{dt} \tag{8}$$

As the right and left arms of model are in parallel, the total stress $\sigma$is given by

$$\sigma=\sigma_1+\sigma_2 \tag{9}$$

Differentiating Eq. (9) we have

$$\frac{d\sigma}{dt}=\frac{d\sigma_1}{dt}+\frac{d\sigma_2}{dt} \tag{10}$$

Eliminating the relations from Eqs. (6 − 8), Eq. (10) becomes

$$\frac{d\sigma}{dt}=\frac{d\varepsilon}{dt}(E_1+E_2)-E_1A\sinh(\sigma-E_2\varepsilon)\alpha \tag{11}$$

For stress relaxation, ε = constant, $\therefore\frac{d\varepsilon}{dt}=0$, from Eq. (11) it can be deduced that

$$\sigma(t)=E_2\varepsilon+\frac{2}{\alpha}\tanh^{-1}\left\{e^{-\alpha E_1 At}\tanh\left(\frac{E_1\varepsilon\alpha}{2}\right)\right\} \tag{12}$$

at t=0, $\sigma(0)=(E_1+E_2)\varepsilon$; and at t=$\infty$, $\sigma(\infty)=E_2\varepsilon$.

For creep, σ= constant, $\therefore\frac{d\sigma}{dt}=0$, from Eq. (11) it can be shown that:

$$\varepsilon(t)=\frac{\sigma}{E_2}-\frac{2}{\alpha E_2}\tanh^{-1}\left\{e^{-\frac{\alpha E_1E_2At}{E_1+E_2}}\tanh\left(\frac{\sigma-E_2\varepsilon(0)}{2}\right)\right\} \tag{13}$$

At *t* = 0, $\varepsilon(0)=\frac{\sigma}{(E_1+E_2)}$ and at t=∞, $\varepsilon(\infty)=\frac{\sigma}{E_2}$.

Eqs. (12 − 13) represent the expressions of stress relaxation and creep respectively as a function of time for the Eyring's model.

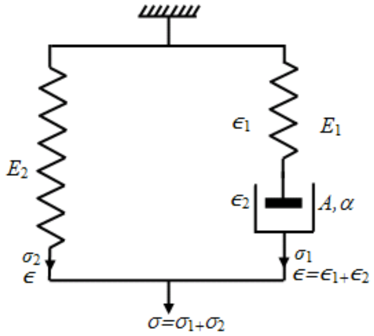

Figure 1: Eyring's three elements visco-elastic model.

3. Material and Methods

Polyester–wool–lycra yarn having 36 tex nominal count spun from Siro spinning technology were used in this study. Stress relaxation phenomenon of these yarns was observed by holding each specimen between two jaws with an initial separation of 200 mm in the Instron tensile tester at a pretension of 0.5 cN/tex. Each specimen was then extended up to a strain level of 15% by moving the upper jaw and was constrained to remain at that strained condition by stopping the upper jaw. The stress values were recorded over 1h at regular intervals. The measurement of creep of all yarns was carried out on a specially designed set up by suspending a 200 mm length of sample from a hook fixed to a wooden stand. Each sample was given a pretension of 0.5 cN/tex by a paperclip. After taking the initial reading, a predetermined load equal to 60% of the average breaking load of each yarn was suspended from the free end of the sample. The extension of the sample was measured by a travelling microscope at different intervals of time starting from 30 s onward till 1 h. Least square method of curve fitting using genetic algorithm on the experimental stress relaxation data with its theoretical model computes the optimum values of the constants for the Eyring's model. The predicted creep curves are then constructed using the creep equation with the constants values obtained from the stress relaxation experiment.

4. Results and Discussion

The experimental and fitted stress relaxation curves obtained with Eq. (12) for P-W-L yarn is depicted in Figs 2. The experimental curve is shown by the solid lines and the corresponding fitted curve is shown by the dotted line. Invariably for each case, a high degree of coefficient of determination (R^2) justifies a good fit to the experimental data. The creep curves are then constructed with Eq. (13) using the values of four constants obtained from the stress relaxation experiment. The experimental creep curve along with those predicted by the Eyring's model for the yarn is illustrated in Fig 3. High values of R^2 substantiate the fact that the Eyring's model is able to predict the yarn creep curves reasonably well.

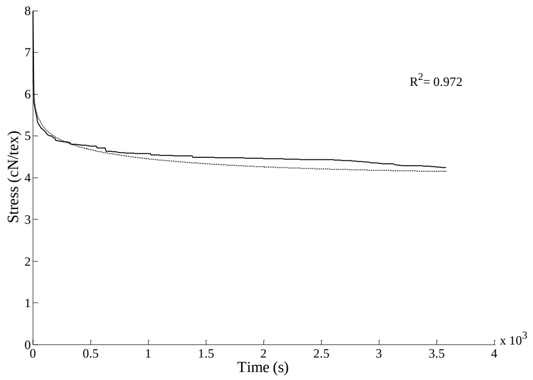

Figure 2: Experimental and fitted stress relaxation curves for P-W-L yarn.

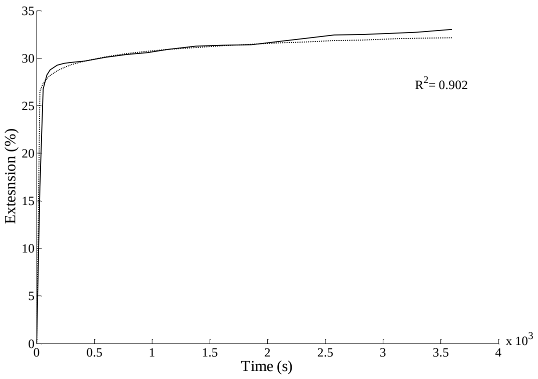

Figure 3: Experimental and fitted creep curves for P-W-L yarn.

5. Conclusions

The stress relaxation and creep behaviours of the P-W-L yarn have been described using Eyring's nonlinear visco-elastic model. The advent of non-traditional search based optimization technique such as GA makes it easy to solve the complex curve fitting problem with the Eyring's equations. Eyring's model provides a common basis to simulate both stress relaxation as well as creep behaviours of worsted core spun yarns with reasonable degree of accuracy.

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